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# **Taxation and strategic reaction: A comparison of Cournot, Stackelberg and collusion**

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# Taxation and strategic reaction: a comparison of Cournot, Stackelberg and collusion

**Abstract:** We study the effect of distortionary taxes on three types of market structure: Cournot duopoly, Stackelberg duopoly, and a monopoly under a collusive agreement between the two rival firms in the industry. We investigate different tax regimes such as a per unit tax, an ad valorem tax and a tax on total revenue. A unit tax rate reduces optimal output and profits for firms while market price rises with the imposition of the tax. Interestingly, the optimal tax rate is the same for all three market structures. The ad valorem tax is imposed on the value of the product and is mostly borne by the Stackelberg follower who ends up producing a greater output than what he would produce in the absence of a tax. The ad valorem tax increases firm output and reduces market price. The total revenue decreases output and increases industry price like the unit tax.

**Keywords:** Cournot duopoly, Stackelberg game, optimal tax rate, Lerner index

**JEL codes:** D42, D43, H21, L12, L13

## 1. Introduction

In the standard Cournot model two identical firms compete on quantities and choose their optimal output levels simultaneously. Cournot (1838) constructed profit functions and used partial differentiation to come up with the firm's best "reaction function" for given output levels of the other firm. The intersection of the two reaction functions results in a stable equilibrium. Stackelberg (1934) came up with a hierarchical model. There is a leader who makes the first move and a follower who takes the residual market demand. The leader has a crucial advantage in that he chooses a quantity that maximizes his payoff, by anticipating the follower's reaction. By using backward induction in this sequential game, Stackelberg (1934) first found the reaction function of the follower, which is a function of the output level of the leader. Then he used it to calculate the reaction function of the leader.

Some previous studies discuss market structure in relation to taxation. Haworth (1998) examines the effect of a specific and ad valorem tax on firm market share in a duopoly where firms have different costs. Haworth (1998) relates tax to cost efficiency and finds that specific and ad valorem commodity taxation increases the market share of the lower-cost firm, while decreasing that of the higher-cost firm. Anderson et al. (2001) present a similar analysis showing that ad valorem taxes are more efficient than unit taxes in the short run with symmetric costs across firms. Delipalla and Keen (1992) demonstrate that ad valorem taxes have welfare advantages over unit taxes for symmetric Cournot-Nash oligopolies. Skeath and Trandel (1994) demonstrate that in the case of monopoly ad valorem tax yields higher consumer surplus, profits and tax revenue. Anderson et al. (2001) contrast the Cournot model of homogeneous products with Bertrand competition of differentiated products and find that unit tax is more efficient under Bertrand competition. Anderson et al. (2001) find that in the Cournot model where products are homogeneous asymmetric cost structures support ad valorem taxes. However, with differentiated products, unit tax may be preferable. In a strategic market game Grazzini (2006) demonstrates that per unit taxation is welfare superior to ad valorem tax if the number of consumers is sufficiently high compared to the number of oligopolists. Opposite to Colombo

and Labrecciosa (2013), Azacis and Collie (2018) show that the choice of tax does not matter in Cournot oligopoly with homogeneous products and general demand functions. Azacis and Collie (2018) also find that tax revenue is always higher with an ad valorem tax than with a specific one.

Some more general studies involve Stern (1987) and Besley (1989) who demonstrate how commodity taxation affects prices, output and profits in imperfectly competitive industries. More specifically, Stern analyses the effect of specific and ad valorem taxation on oligopolistic and monopolistically competitive markets. Schröder (2004) and Vetter (2013) discuss ad valorem and specific tax in the context of monopolistic competition.

Our study is standard in that we investigate a linear demand curve and a market shared by two firms with identical, constant marginal cost. In this sense, we use a very simple framework to demonstrate the effect of different types of tax on market structure. Furthermore, we analyze three types of tax, a per-unit tax, an ad valorem tax and a tax on total revenue. Ad valorem taxes are imposed on the value of the good, while a specific tax is essentially levied on the units of the good produced by firms. Our findings are consistent with Haworth (1998) if marginal cost is assumed to be equal for the two firms. However, Howarth (1998) puts the discussion in the context of a simple oligopoly with no reference to other market structures. We analyze the duopolists under a cartel agreement and a Stackelberg sequential game in addition to Cournot. We compute the effect of these three types of tax on the three market structures in terms of firm output, profit and industry price. We investigate optimal tax rate for unit and ad valorem tax from the perspective of tax collection to the government since the government may have incentives to maintain a given market structure in order to maximize its tax revenue. We also compute the Lerner index of the three market structures under the three tax regimes, that is, a total of nine outcomes.

The paper has several sections. In section 2 we demonstrate the case without taxation. Section 3 presents the case of a unit tax. Section 4 covers the ad valorem tax as a tax on the value of the good. Section 5 reveals the effect of a tax on total revenue. The paper ends with conclusions.

## 2. The case without taxation

This initial case summarizes the output, price and profit level of each market structure in the absence of taxation. This case serves as a benchmark for the other cases when tax is levied. We follow the standard Cournot and Stackelberg setting where the Cournot outcome is the result of a simultaneous, one-stage game, while the Stackelberg one follows from a sequential, dynamic game. The Cournot case represents a symmetrical duopoly, while the Stackelberg duopoly is one of a leader and a follower. We also show a monopoly under a collusive agreement between the two rival firms in the industry. We assume that a homogenous product is produced and both firms have a constant marginal cost of production  $c$ . The market demand and the cost function of each firm are given as

$$p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad i = 1, 2 \quad (1)$$

Using simple optimization, we find the reaction function of each of the two duopolists in the Cournot case.

$$\pi_1(q_1) = pq_1 - cq_1 \quad (2)$$

$$\pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1 \quad (3)$$

$$\frac{d\pi_1}{dq_1} = a - 2bq_1 - bq_2 - c = 0 \quad (4)$$

$$q_1 = \frac{a - bq_2 - c}{2b}, \quad (5)$$

and symmetrically for the second Cournot duopolist,

$$q_2 = \frac{a - bq_1 - c}{2b} \quad (6)$$

produces the following optimal quantities for the two identical Cournot duopolists, industry price and profits for the two firms.

$$q_1^c = \frac{a - c}{3b} = q_2^c \quad q = q_1^c + q_2^c = \frac{2(a - c)}{3b} \quad (7)$$

$$p = a - bq = a - \frac{2b(a - c)}{3b} = \frac{a + 2c}{3} \quad (8)$$

$$\pi_1 = (p - c)q_1 = \left(\frac{a + 2c}{3} - c\right)\frac{(a - c)}{3b} = \frac{(a - c)^2}{9b} = \pi_2 \quad (9)$$

In the case of monopoly, we have one seller since the two Cournot duopolists collusively agree to produce the monopoly output.

$$\pi(q) = pq - cq \quad p(q) = a - bq \quad (10)$$

$$\pi = (a - bq - c)q = (a - c)q - bq^2 \quad (11)$$

$$\frac{d\pi}{dq} = a - c - 2bq = 0 \quad (12)$$

$$q_m = \frac{a - c}{2b}, \quad (13)$$

where each Cournot duopolist would ideally produce half of this output under collusion, that is,  $\frac{q_m}{2} = \frac{a - c}{4b}$ . Apparently, this output is lower than their output without collusion, i.e.,

$q_1^c = \frac{a - c}{3b} = q_2^c$ . Furthermore, with monopoly we obtain a higher price and industry profit.

$$p_m = a - \frac{b(a - c)}{2b} = \frac{a + c}{2}, \text{ and} \quad (14)$$

$$\pi_m = (p - c)q = \left(\frac{a + c}{2} - c\right)\frac{(a - c)}{2b} = \frac{(a - c)^2}{4b} \quad (15)$$

Under Stackelberg the follower optimizes along his residual demand curve, given the output and behavior of the leader.

$$p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad i = 1, 2 \quad (16)$$

$$\pi_2(q_2) = pq_2 - cq_2 \quad (17)$$

$$\pi_2 = (a - bq_1 - bq_2)q_2 - cq_2 = aq_2 - bq_1q_2 - bq_2^2 - cq_2 \quad (18)$$

$$\frac{d\pi_2}{dq_2} = a - bq_1 - 2bq_2 - c = 0 \quad (19)$$

$$q_2 = \frac{a - bq_1 - c}{2b}, \quad (20)$$

The profit of the leader is

$$\pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1 \quad (21)$$

Substituting for  $q_2$  in the profit function of the incumbent,

$$\pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 = aq_1 - bq_1^2 - \frac{bq_1(a - bq_1 - c)}{2b} - cq_1 = \frac{aq_1 - bq_1^2 - cq_1}{2} \quad (22)$$

$$\frac{d\pi_1}{dq_1} = \frac{a - c}{2} - bq_1 = 0 \quad (23)$$

$$q_1^s = \frac{a - c}{2b} \quad (24)$$

gives the output of the leader, while that of the follower is

$$q_2^s = \frac{a - c}{4b}, \text{ or,} \quad (25)$$

the leader produces twice as much as the follower in a Stackelberg sequential game. The total industry output exceeds that of the Cournot duopolists at  $q_1^s + q_2^s = \frac{3(a - c)}{4b}$ .

$$p = a - b(q_1^s + q_2^s) = a - \frac{b(a - c)}{2b} - \frac{b(a - c)}{4b} = a - \frac{3(a - c)}{4} = \frac{a + 3c}{4} \quad (26)$$

Profits are, respectively,

$$\pi_1 = pq_1 - cq_1 = \left(\frac{a + 3c}{4} - c\right) \frac{(a - c)}{2b} = \frac{(a - c)^2}{8b} \quad (27)$$

$$\pi_2 = pq_2 - cq_2 = \left(\frac{a + 3c}{4} - c\right) \frac{(a - c)}{4b} = \frac{(a - c)^2}{16b} \quad (28)$$

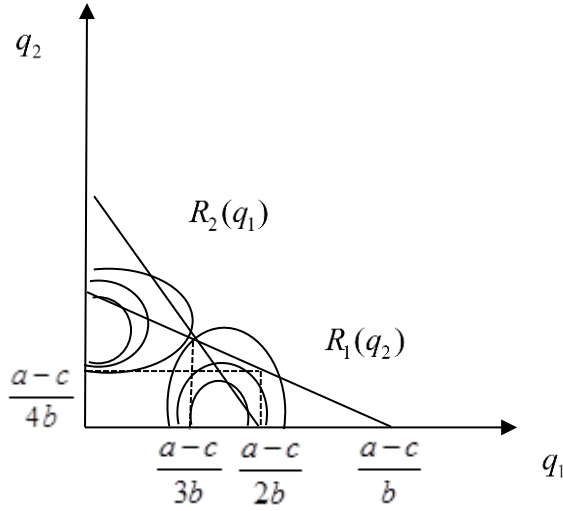


Figure 1. Cournot equilibrium versus Stackelberg

Figure 1 illustrates how the equilibrium is shifted from the Cournot outcome to Stackelberg. Whereas two symmetrical oligopolists produce identical output  $\frac{a-c}{3b}$  at the crossing point of their reaction functions, in the Stackelberg game the leader has a first-mover advantage and produces twice as much as the follower along his reaction function.

### 3. The case of a per-unit tax

If the government imposes a unit tax to the amount  $t$ , then total tax collection is  $T = tq$  where  $q = q_1 + q_2$  is cumulative industry output. A unit tax acts like an extra cost to the firm. In the Cournot case profit maximization gives the optimal output levels of the Cournot duopolists.

$$\pi_1(q_1) = (p - c - t)q_1 \quad (29)$$

$$\pi_1 = (a - bq_1 - bq_2 - c - t)q_1 = (a - c - t)q_1 - bq_1^2 - bq_1q_2 \quad (30)$$

$$\frac{d\pi_1}{dq_1} = (a - c - t) - 2bq_1 - bq_2 = 0, \quad (31)$$

and identically for the second duopolist,

$$\frac{d\pi_2}{dq_2} = (a - c - t) - 2bq_2 - bq_1 = 0 \quad (32)$$

gives the optimal output levels

$$q_1^c = \frac{a - c - t}{3b} = q_2^c \quad (33)$$

Furthermore, the industry price and firm profits with the unit tax are

$$p_t = a - bq = a - \frac{2b(a - c - t)}{3b} = \frac{a + 2c + 2t}{3} \quad (34)$$

$$\pi_1(q_1) = (p - c - t)q_1 = \left(\frac{a + 2c + 2t}{3} - c - t\right) \frac{(a - c - t)}{3b} = \frac{(a - c - t)^2}{9b} = \pi_2 \quad (35)$$

As could be expected, the output and profit levels of the two Cournot duopolists fall with the introduction of the unit tax with profits falling very quickly with a higher unit tax rate since the effect of the tax rate comes under a square. At the same time, the industry price increases with the imposition of the tax. A comparison of the results on quantity, price and profit confirms their validity – in the case of a zero tax the results are identical to those without a tax. Figure 2 shows the effect of the unit tax on the reaction functions of the two duopolists. The tax lowers the output levels of each Cournot duopolist and moves both reaction functions to the left. This results in a new equilibrium where both duopolists produce less at the optimum.

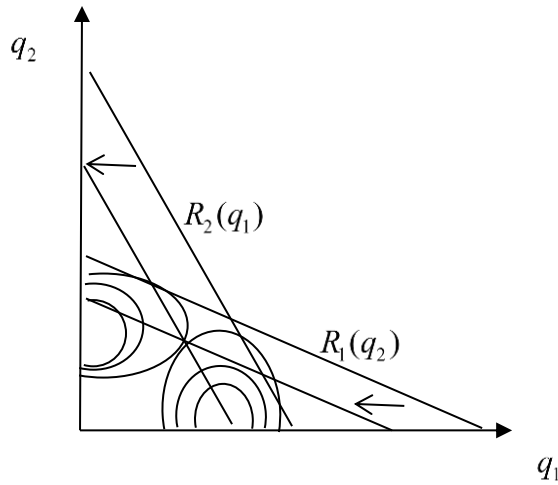


Figure 2. Effect of tax on the reaction functions of the Cournot duopolists

Under a cartel agreement the tax will again act as an additional cost to the joint production of the firms.

$$\pi(q) = (p - c - t)q \quad p(q) = a - bq \quad (36)$$

$$\pi(q) = (a - bq - c - t)q = (a - c - t)q - bq^2 \quad (37)$$

$$\frac{d\pi}{dq} = a - c - t - 2bq = 0 \quad (38)$$

$$q_m^t = \frac{a - c - t}{2b} \quad (39)$$

$$p_m^t = a - \frac{b(a - c - t)}{2b} = \frac{a + c + t}{2} \quad (40)$$

$$\pi_m^t = \left(\frac{a + c + t}{2} - c - t\right) \frac{(a - c - t)}{2b} = \frac{(a - c - t)^2}{4b} \quad (41)$$

Again, these results confirm the theory in the absence of a tax. The output, price and profit of the monopolist would be equal to those in the case of zero tax. The presence of the tax reduces the output and the profit, with profit being reduced much more significantly since the tax rate

$t$  is in the square and carries a negative sign. This implies that increases in the tax rate by even small amounts provoke large decreases in the profit level of the cartel. Comparing the after-tax profits of the cartel with the joint profit of the Cournot duopolists, we obtain

$$\pi_m^t > \pi_1^C + \pi_2^C, \text{ since} \quad (42)$$

$$\frac{(a-c-t)^2}{4b} > \frac{2(a-c-t)^2}{9b} \quad (43)$$

Even with the imposition of a unit tax on quantity, the two Cournot duopolists are better off colluding. What would the effect of a unit tax be on the two duopolists under Stackelberg?

$$p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad i = 1, 2 \quad (44)$$

$$\pi_2(q_2) = pq_2 - cq_2 - tq_2 \quad (45)$$

$$\pi_2 = (a - bq_1 - bq_2)q_2 - cq_2 - tq_2 = aq_2 - bq_1q_2 - bq_2^2 - cq_2 - tq_2 \quad (46)$$

$$\frac{d\pi_2}{dq_2} = a - bq_1 - 2bq_2 - c - t = 0 \quad (47)$$

$$q_2 = \frac{a - bq_1 - c - t}{2b}, \quad (48)$$

The profit of the leader is

$$\pi_1 = (a - bq_1 - bq_2)q_1 - cq_1 - tq_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1 - tq_1 \quad (49)$$

Substituting for  $q_2$  in the profit function of the incumbent,

$$\begin{aligned} \pi_1 &= (a - bq_1 - bq_2)q_1 - cq_1 - tq_1 = aq_1 - bq_1^2 - \frac{bq_1(a - bq_1 - c - t)}{2b} - cq_1 - tq_1 = \\ &= \frac{aq_1 - bq_1^2 - cq_1 - tq_1}{2} \end{aligned} \quad (50)$$

$$\frac{d\pi_1}{dq_1} = \frac{a - c - t}{2} - bq_1 = 0 \quad (51)$$

$$q_1^S = \frac{a - c - t}{2b} \quad (52)$$

gives the output of the leader, while that of the follower is

$$q_2^S = \frac{a - c - t}{4b}, \text{ or,} \quad (53)$$

the leader produces twice as much as the follower in a Stackelberg game even with a unit tax rate. To compute industry price,

$$p = a - b(q_1^S + q_2^S) = a - \frac{b(a - c - t)}{2b} - \frac{b(a - c - t)}{4b} = a - \frac{3(a - c - t)}{4} = \frac{a + 3c + 3t}{4} \quad (54)$$



While industry output decreases, industry price increases with tax. The profits of the incumbent and the entrant are, respectively,

$$\pi_1 = pq_1 - cq_1 - tq_1 = \left(\frac{a+3c+3t}{4} - c - t\right) \frac{(a-c-t)}{2b} = \frac{(a-c-t)^2}{8b} \quad (55)$$

$$\pi_2 = pq_2 - cq_2 - tq_2 = \left(\frac{a+3c+3t}{4} - c - t\right) \frac{(a-c-t)}{4b} = \frac{(a-c-t)^2}{16b} \quad (56)$$

The entrant receives half of the profit of the incumbent, as in the case without tax. The tax significantly decreases the profit to both players. We see that in comparison with Cournot Stackelberg has a higher output level, lower market price and lower industry profit, although the Stackelberg leader is still better off than the Cournot duopolist. This comes at the expense of the Stackelberg follower. Table 1 summarizes the results of optimal output, industry price and firm profit under the different market structures. The revenue to the government can be expressed in all three cases. In the Cournot case,

$$T = tq = t(q_1 + q_2) = \frac{t2(a-c-t)}{3b} = \frac{2(a-c)t - 2t^2}{3b} \quad (57)$$

$$\frac{dT}{dt} = 2(a-c) - 4t = 0 \quad (58)$$

$$t^* = \frac{a-c}{2} \quad (59)$$

From the perspective of total tax collection in the Cournot case and tax revenue being maximized, the value of  $t^*$  gives optimal tax rate to the government. With monopoly,

$$T = tq_m^t = \frac{t(a-c-t)}{2b} = \frac{(a-c)t - t^2}{2b} \quad (60)$$

$$\frac{dT}{dt} = a - c - 2t = 0 \quad (61)$$

$$t^* = \frac{a-c}{2} \quad (62)$$

To find optimal tax rate in Stackelberg,

$$T = tq = t(q_1 + q_2) = t\left(\frac{a-c-t}{2b} + \frac{a-c-t}{4b}\right) = \frac{3t(a-c-t)}{4b} = \frac{3(at-ct-t^2)}{4b} \quad (63)$$

$$\frac{dT}{dt} = a - c - 2t = 0 \quad (64)$$

$$t^* = \frac{a-c}{2} \quad (65)$$

We see that the same optimal tax rate on quantity obtains in all three cases, Cournot, collusion and Stackelberg. An explanation is that we have a tax based on the amount of output produced.

Although the unit tax changes the total output levels in all cases, it does not change the output distribution between the duopolists. The Cournot duopolists are still producing symmetrical levels of output, and the leader is again producing twice as much the follower. Table 2 presents these optimal tax rates.

#### 4. The case of an ad valorem tax

An ad valorem tax  $t$  is imposed on the value of the product. It would be imposed by governments when the value of the product is significant. The ad valorem tax acts like some quantity is taken away from the firm in the form of tax to the government. Thus, the profit to the firm becomes

$$\pi_1(q_1) = p(q_1 - t) - cq_1 \quad p = a - bq_1 - bq_2 \quad (66)$$

Hence, the tax collected by the government is like a loss to the firm to the amount of  $-pt$ . In the Cournot case,

$$\pi_1 = (a - bq_1 - bq_2)(q_1 - t) - cq_1 \quad (67)$$

$$\frac{d\pi_1}{dq_1} = a - bq_1 - bq_2 - b(q_1 - t) - c = 0 \quad (68)$$

$$a - 2bq_1 - bq_2 + bt - c = 0 \quad (69)$$

$$q_1 = \frac{a - bq_2 + bt - c}{2b}, \quad (70)$$

and symmetrically for the second Cournot duopolist,

$$q_2 = \frac{a - bq_1 + bt - c}{2b} \quad (71)$$

produces equal optimal quantities for the Cournot duopolists

$$q_1^c = \frac{a + bt - c}{3b} = q_2^c \quad q = q_1^c + q_2^c = \frac{2(a + bt - c)}{3b} \quad (72)$$

The Cournot industry price and profit levels are, respectively,

$$p_c = a - bq = a - \frac{2b(a + bt - c)}{3b} = \frac{a - 2bt + 2c}{3} \quad (73)$$

$$\begin{aligned} \pi_1 &= p(q_1 - t) - cq_1 = \frac{(a - 2bt + 2c)}{3} \left( \frac{a + bt - c}{3b} - t \right) - c \frac{(a + bt - c)}{3b} = \\ &= \frac{(a - c)^2 - bt(4a - 4bt + 5c)}{9b} = \pi_2 \end{aligned} \quad (74)$$

We notice that in the absence of an ad valorem tax the profit of each firm is equal to the previously obtained profit of each Cournot oligopolist. In the case of monopoly, the ad valorem tax gives the following results.

$$\pi(q) = p(q-t) - cq \quad p(q) = a - bq \quad (75)$$

$$\pi = (a - bq)(q - t) - cq = aq - at - bq^2 + bqt \quad (76)$$

$$\frac{d\pi}{dq} = a - 2bq + bt - c = 0 \quad (77)$$

$$q_m = \frac{a + bt - c}{2b}, \quad (78)$$

$$p_m = a - \frac{b(a + bt - c)}{2b} = \frac{a - bt + c}{2}, \text{ and} \quad (79)$$

$$\begin{aligned} \pi_m &= \frac{(a - bt + c)}{2} \left( \frac{a + bt - c}{2b} - t \right) - \frac{c(a + bt - c)}{2b} = \frac{(a - bt)^2 - c^2 - 2c(a + bt - c)}{4b} = \\ &= \frac{(a - c)^2 - bt(4a - bt + 2c)}{4b} \end{aligned} \quad (80)$$

which under zero tax or  $t = 0$  gives the same profit as under monopoly. Under Stackelberg with an ad valorem tax the profit functions of the follower could be expressed as,

$$\pi_2(q_2) = p(q_2 - t) - cq_2 \quad p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad (81)$$

$$\pi_2 = (a - bq_1 - bq_2)(q_2 - t) - cq_2 \quad (82)$$

$$\frac{d\pi_2}{dq_2} = a - bq_1 - 2bq_2 + bt - c = 0 \quad (83)$$

$$q_2 = \frac{a - bq_1 + bt - c}{2b}, \quad (84)$$

The profit of the leader, upon substitution for  $q_2$ , is

$$\pi_1 = (a - bq_1 - bq_2)(q_1 - t) - cq_1 = \frac{(a - bq_1 - bt + c)(q_1 - t)}{2} - cq_1 \quad (85)$$

$$\frac{d\pi_1}{dq_1} = -\frac{b(q_1 - t)}{2} + \frac{a - bq_1 - bt + c}{2} - c = 0 \quad (86)$$

$$q_1^s = \frac{a - c}{2b} \quad (87)$$

gives the output of the leader. The result is interesting in that the output of the Stackelberg leader is not dependent on the ad valorem tax and remains unchanged. For the follower,

$$q_2^s = \frac{a - c + 2bt}{4b} \quad (88)$$

This result indicates that the ad valorem tax is mostly borne by the Stackelberg follower who ends up producing a greater output than what he would produce in the absence of a tax. Under the unit tax rate both competitors were found to produce an amount smaller than that without

tax. This can be explained by the fact that the ad valorem tax is imposed on the value of the produce and not its volume. Both Stackelberg competitors would be affected negatively through price, which decreases due to the tax.

$$p = a - b(q_1^S + q_2^S) = a - \frac{b(a-c)}{2b} - \frac{b(a-c+2bt)}{4b} = \frac{a+3c-2bt}{4} \quad (89)$$

Profits are, respectively,

$$\begin{aligned} \pi_1 &= p(q_1 - t) - cq_1 = \frac{(a+3c-2bt)}{4} \frac{(a-c-2bt)}{2b} - \frac{c(a-c)}{2b} = \\ &= \frac{(a-2bt)^2 - c^2 + 2ac - 2c^2 - 4bct - 4ac + 4c^2}{8b} = \frac{(a-c)^2 - 4bt(a-bt+c)}{8b} \end{aligned} \quad (90)$$

$$\begin{aligned} \pi_2 &= p(q_2 - t) - cq_2 = \frac{(a+3c-2bt)}{4} \frac{(a-c-2bt)}{4b} - \frac{c(a-c+2bt)}{4b} = \\ &= \frac{(a-c)^2 - 4bt(a-bt+3c)}{16b} \end{aligned} \quad (91)$$

As in the case of unit tax, we can compute the optimal tax rate of an ad valorem tax which maximizes total tax collection to the government. We do this in all three situations, a Cournot duopoly, monopoly and Stackelberg. Since the revenues which the government receives are a deduction of the firms' profits to the amount of  $pt$ , this gives the following results for the optimal ad valorem tax to the three types of market structure.

$$T = p_c t = \frac{(a-2bt+2c)t}{3} = \frac{at-2bt^2+2ct}{3} \quad (92)$$

$$\frac{dT}{dt} = a - 4bt + 2c = 0 \quad (93)$$

$$t^* = \frac{a+2c}{4b} \quad (94)$$

The value of  $t^*$  gives optimal tax rate of an ad valorem tax in the Cournot case. With monopoly,

$$T = p_m t = \frac{t(a-bt+c)}{2} = \frac{at-bt^2+ct}{2} \quad (95)$$

$$\frac{dT}{dt} = a - 2bt + c = 0 \quad (96)$$

$$t^* = \frac{a+c}{2b} \quad (97)$$

gives the same optimal tax rate under ad valorem tax in the case of monopoly. Finally, for the Stackelberg game we obtain

$$T = p_s t = \frac{t(a-2bt+3c)}{4} = \frac{at-2bt^2+3ct}{4} \quad (98)$$

$$\frac{dT}{dt} = a - 4bt + 3c = 0 \quad (99)$$

$$t^* = \frac{a + 3c}{4b} \quad (100)$$

We find that the optimal tax rate of monopoly exceeds that of Cournot which is possibly the result of a higher value (price) under collusion. The optimal tax rate under Stackelberg also exceeds that of Cournot. Table 2 summarizes these results.

## 5. The case of a tax on total revenue

The tax on the total revenue will be a percentage of the total revenue, that is,  $t \in [0,1)$ . When the government imposes a tax on revenue some of it is taken away from the firm such that

$$\pi_1(q_1) = pq_1(1-t) - cq_1 \quad p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad (101)$$

$$\pi_2(q_2) = pq_2(1-t) - cq_2 \quad (102)$$

$$\pi_1 = (a - bq_1 - bq_2)q_1(1-t) - cq_1 \quad (103)$$

$$\frac{d\pi_1}{dq_1} = (1-t)(a - 2bq_1 - bq_2) - c = 0 \quad (104)$$

where under Cournot equilibrium the outputs of the two Cournot duopolists are equal, that is, for firm 1,

$$(1-t)(a - 3bq_1) - c = 0 \quad (105)$$

$$a - 3bq_1 = \frac{c}{1-t} \quad (106)$$

$$q_1^C = \frac{a - at - c}{3b(1-t)} = q_2^C, \text{ and} \quad (107)$$

$$p_C = \frac{a - at + 2c}{3(1-t)} \quad (108)$$

The profit of each duopolist is

$$\pi_1 = q_1(p - pt - c) = \frac{(a - at - c)}{3b(1-t)} \left( \frac{a - at + 2c}{3} - c \right) = \frac{(a - at - c)^2}{9b(1-t)} = \pi_2 \quad (109)$$

A more thorough investigation shows that the tax on total revenue increases the output and reduces the industry price for the Cournot duopolists. With monopoly the tax on total revenue takes away an amount  $-pqt$  from the firm so that the profit becomes

$$\pi(q) = pq(1-t) - cq \quad p(q) = a - bq \quad a, b > 0 \quad (110)$$

$$\pi(q) = (a - bq)q(1-t) - cq = (aq - bq^2)(1-t) - cq \quad (111)$$

$$\frac{d\pi}{dq} = (1-t)(a-2bq) - c = 0 \quad (112)$$

$$a-2bq = \frac{c}{1-t} \quad (113)$$

$$q_m = \frac{a-at-c}{2b(1-t)}, \text{ and} \quad (114)$$

$$p_m = a - \frac{a-at-c}{2(1-t)} = \frac{a-at+c}{2(1-t)} \quad (115)$$

The profit of the monopolist is

$$\pi_m = pq(1-t) - cq = \frac{(a-at+c)(a-at-c)}{4b(1-t)} - \frac{c(a-at-c)}{2b(1-t)} = \frac{(a-at-c)^2}{4b(1-t)} \quad (116)$$

With Stackelberg the optimization problem for the incumbent and the entrant is

$$\pi_1(q_1) = pq_1(1-t) - cq_1 \quad p(q) = a - b(q_1 + q_2) \quad a, b > 0 \quad C_i = cq_i \quad (117)$$

$$\pi_2(q_2) = pq_2(1-t) - cq_2 \quad (118)$$

$$\pi_2 = (a - bq_1 - bq_2)q_2(1-t) - cq_2 = (aq_2 - bq_1q_2 - bq_2^2)(1-t) - cq_2 \quad (119)$$

$$\frac{d\pi_2}{dq_2} = (a - bq_1 - 2bq_2)(1-t) - c = 0 \quad (120)$$

$$a - bq_1 - 2bq_2 = \frac{c}{(1-t)} \quad (121)$$

$$q_2^s = \frac{(a - bq_1)(1-t) - c}{2b(1-t)} \quad (122)$$

Substituting for  $q_2$  in the profit function of the incumbent,

$$\begin{aligned} \pi_1 &= (a - bq_1 - bq_2)q_1(1-t) - cq_1 = \left[ a - bq_1 - \frac{(a - bq_1)(1-t) - c}{2(1-t)} \right] q_1(1-t) - cq_1 = \\ &= \left[ \frac{a - bq_1}{2} + \frac{c}{2(1-t)} \right] q_1(1-t) - cq_1 = \left[ \frac{(a - bq_1)(1-t) + c}{2} \right] q_1 - cq_1 = \\ &= \frac{(a - bq_1)(1-t)q_1}{2} - \frac{cq_1}{2} \end{aligned} \quad (123)$$

which gives the optimal output for the incumbent

$$\frac{d\pi_1}{dq_1} = (a - bq_1)(1-t) - b(1-t)q_1 - c = 0 \quad (124)$$

$$a(1-t) - 2b(1-t)q_1 - c = 0 \quad (125)$$

$$q_1^s = \frac{a(1-t)-c}{2b(1-t)} = \frac{a-at-c}{2b(1-t)} \quad (126)$$

gives the output of the leader, while that of the follower is

$$q_2^s = \frac{a}{2b} - \frac{a-at-c}{4b(1-t)} - \frac{c}{2b(1-t)} = \frac{a-at-c}{4b(1-t)}, \quad (127)$$

that is, the leader again produces twice as much as the follower. For the industry price,

$$p = a - b(q_1^s + q_2^s) = a - \frac{a-at-c}{2(1-t)} - \frac{a-at-c}{4(1-t)} = \frac{a(1-t)+3c}{4(1-t)} \quad (128)$$

The profits of the incumbent and the entrant are, respectively,

$$\pi_1 = pq_1 - cq_1 - tq_1 = \left[ \frac{a(1-t)}{4} + \frac{3c}{4} - c \right] \left[ \frac{a(1-t)-c}{2b(1-t)} \right] = \frac{(a-at-c)^2}{8b(1-t)} \quad (129)$$

$$\pi_2 = pq_2 - cq_2 - tq_2 = \left[ \frac{a(1-t)}{4} + \frac{3c}{4} - c \right] \left[ \frac{a(1-t)-c}{4b(1-t)} \right] = \frac{(a-at-c)^2}{16b(1-t)} \quad (130)$$

The entrant receives half of the profit of the incumbent, as in the case without tax. The tax significantly decreases the profit to both players. Table 1 again presents these results. It also lists the values of the Lerner index we have computed for the respective market structures under the different tax regimes. In the absence of tax, we have  $L_m > L_c > L_s$ , that is, the degree of market power is highest with monopoly followed by Cournot and Stackelberg. In the case of the unit tax the market power of the Stackelberg firms is lowest because the entry of the follower reduces the price for the Stackelberg leader (it could be checked that  $p_m > p_c > p_s$ ).

## Conclusion

In comparison with Cournot the Stackelberg duopoly has a bigger output, lower price, and lower industry profit. Although the Stackelberg leader has a higher profit than the Cournot duopolist, the Stackelberg follower is in a worse position. It is beneficial for Cournot duopolists to collude even with taxation. The unit tax and the tax on total revenue have a similar effect – in all market structures output and profit levels fall, while market price increases from the pre-tax level. With both types of tax, the Stackelberg leader produces twice as much as the follower and obtains twice his profit. We find that the optimal tax rate from the perspective of total tax collection for the unit tax is the same for the three markets. This could be because the tax is imposed on the amount produced. Although the unit tax changes the total output levels, it does not change the output distribution between the firms.

The effect of the ad valorem tax is opposite to that of the other two types of taxes. The ad valorem tax increases the output levels of firms, while lowering the industry price. The result follows from the fact that the tax is imposed on the value of the product and not on the quantity. The output of the Stackelberg leader is unaffected by the ad valorem tax. The ad valorem tax is mostly borne by the Stackelberg follower who ends up producing more than before the imposition of the tax. Both the incumbent and the entrant are affected negatively through price,

which decreases with the tax. The Lerner index is highest for the monopoly firm, followed by the Cournot duopolists. The market power of the firms in a Stackelberg game is lowest because the entry of the follower reduces the price for the Stackelberg leader.

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